DETERMINANTS & CRAMER’S RULE

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DETERMINANT OF 2×2 MATRIX

The determinant of a 2×2 matrix is the difference of the entries on the diagonal.

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

EVALUATE

Find the determinant of the matrix:

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

Solution:

$$\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} = 1(5) - 2(3) = 5 - 6 = -1$$

DETERMINANT OF 3×3 MATRIX

The determinant of a 3×3 matrix is the difference in the sum of the products in red from the sum of the products in black.

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \left|\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}\right|\begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$$

Determinant = [a(ei)+b(fg)+c(dh)]-[g(ec)+h(fa)+i(db)]

Evaluate Determinant of A

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -2 & 0 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

Solution:

$$|A| = \left| \begin{matrix} 2 & -1 & 3 \\ -2 & 0 & 1 \\ 1 & 2 & 4 \end{matrix} \right| \begin{matrix} 2 & -1 \\ -2 & 0 \\ 1 & 2 \end{matrix}$$

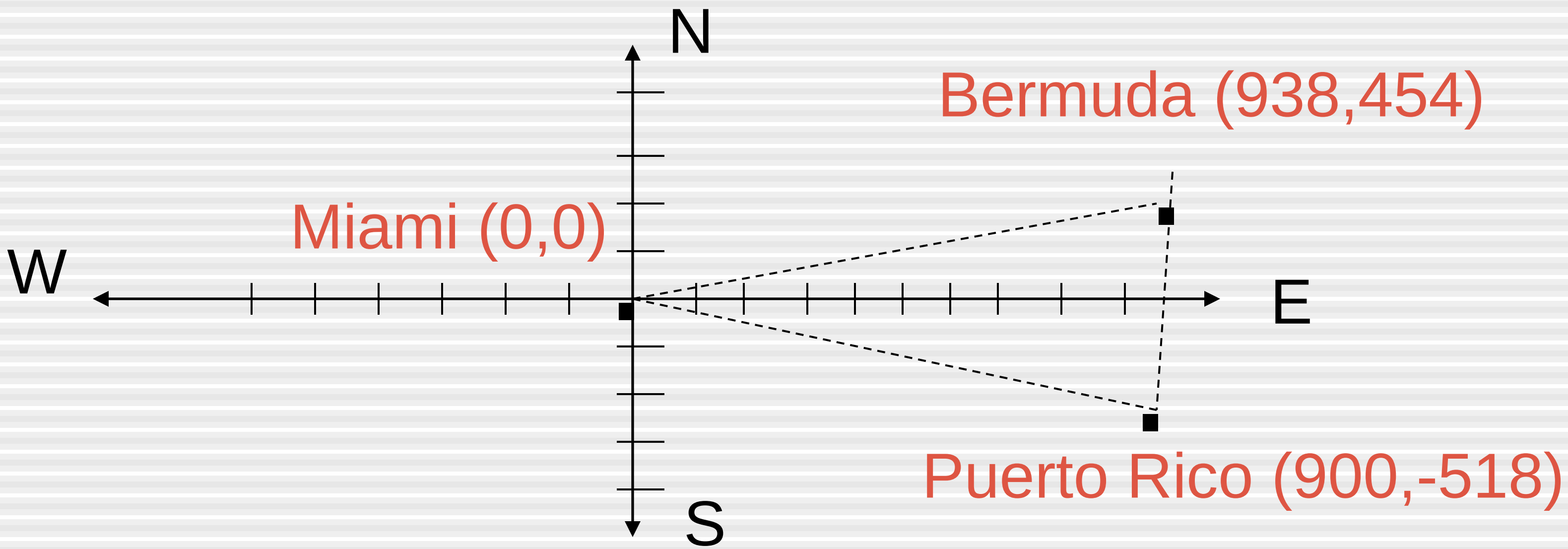
$$= [0 + (-1) + (-12)] - (0 + 4 + 8)$$

$$= -13 - 12$$

$$= -25$$

USING DETERMINANT IN REAL LIFE

The Bermuda Triangle is a large trianglular region in the Atlantic ocean. Many ships and airoplanes have been lost in this region. The triangle is formed by imaginary lines connecting Bermuda, Puerto Rico, and Miami, Florida. Use a determinant to estimate the area of the Bermuda Triangle.



SOLUTION

The approximate coordinates of the Bermuda Triangle's three vertices are: (938,454), (900,-518), and (0,0). So the area of the region is as follows:

$$Area = \pm\frac{1}{2}\begin{vmatrix} 938 & 454 & 1 \\ 900 & -518 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

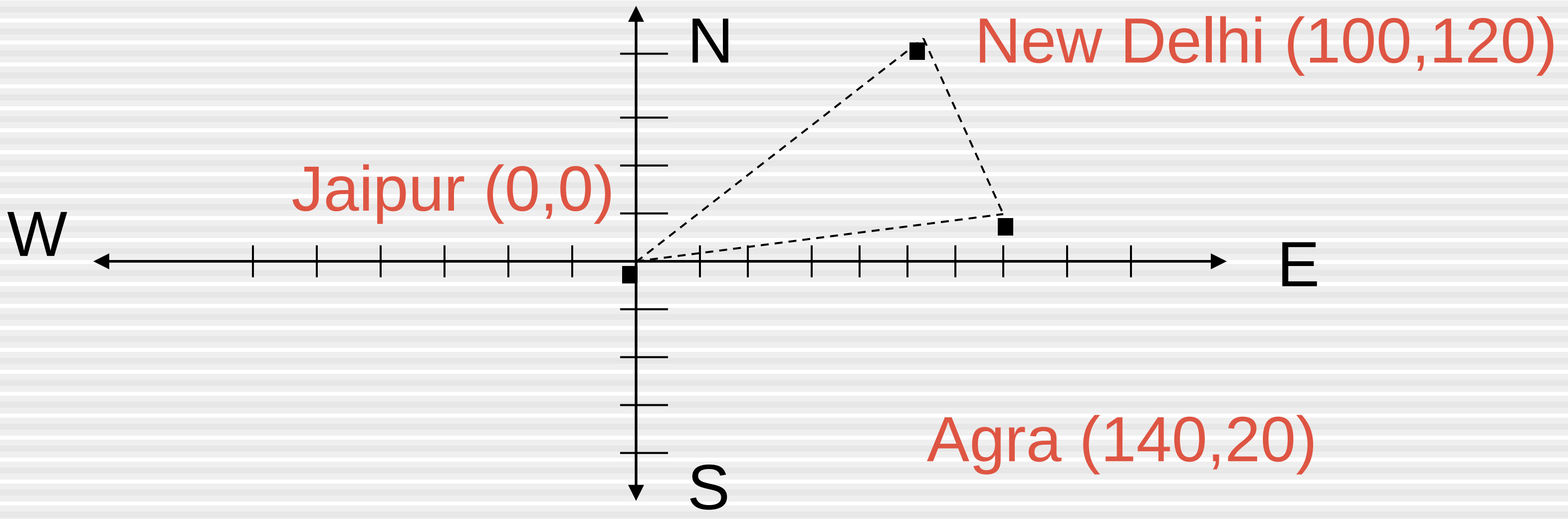
$$Area = \pm\frac{1}{2}[(-458,884+0+0)-(0+0+408,600)]$$

$$Area = 447,242$$

Hence, area of the Bermuda Triangle is about 447,000 square miles.

USING DETERMINANT IN REAL LIFE

The Golden Triangle is a large triangular region in the India.The Taj Mahal is one of the many wonders that lie within the boundaries of this triangle. The triangle is formed by the imaginary lines that connect the cities of New Delhi, Jaipur, and Agra. Use a determinant to estimate the area of the Golden Triangle. The coordinates given are measured in miles.



SOLUTION

The approximate coordinates of the Golden Triangle's three vertices are: (100,120), (140,20), and (0,0). So the area of the region is as follows:

$$Area = \pm\frac{1}{2}\begin{vmatrix} 100 & 120 & 1 \\ 140 & 20 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$Area = \pm\frac{1}{2}[(2000+0+0)-(0+0+16800)]$$

$$Area = 7400$$

Hence, area of the Golden Triangle is about 7400 square miles.

CRAMER"S RULE FOR A 2×2 SYSTEM

Let A be the co-efficient matrix of the linear system: ax+by= e & cx+dy= f.

IF det A ≠0, then the system has exactly one solution. The solution is:

$$x = \frac{\Delta x}{|A|} = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A}$$

$$y = \frac{\Delta y}{|A|} = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$$

The numerators for x and y are the determinant of the matrices formed by using the column of constants as replacements for the coefficients of x and y, respectively.

EXAMPLE

Use cramer’s rule to solve this system:

$$8x+5y = 2$$
$$2x-4y = -10$$

SOLUTION

Solution: Evaluate the determinant of the coefficient matrix

$$\begin{vmatrix} 8 & 5 \\ 2 & -4 \end{vmatrix} = -32 - 10 = -42$$

Apply cramer's rule since the determinant is not zero.

$$x = \frac{\begin{vmatrix} 2 & 5 \\ -10 & -4 \end{vmatrix}}{-42} = \frac{-8-(-50)}{-42} = \frac{42}{-42} = -1$$

$$y = \frac{\begin{vmatrix} 8 & 2 \\ 2 & -10 \end{vmatrix}}{-42} = \frac{-80-4}{-42} = \frac{-84}{-42} = 2$$

The solution is (-1,2)

CRAMER”S RULE FOR A 3×3 SYSTEM

Let A be the co-efficient matrix of the linear system:
ax+by+cz= j, dx+ey+fz= k, and gx+hy+iz=l.

IF det A ≠0, then the system has exactly one solution.
The solution is:

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A}, y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A}, z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}$$

EXAMPLE

The atomic weights of three compounds are shown. Use a linear system and Cramer's rule to find the atomic weights of carbon(C), hydrogen(H), and oxygen(O).

Compound	Formula	Atomic weight
Methane	CH_4	16
Glycerol	$C_3H_8O_3$	92
Water	H_2O	18

SOLUTION

Write a linear system using the formula for each compound

$$\begin{aligned} C + 4H &= 16 \\ 3C + 8H + 3O &= 92 \\ 2H + O &= 18 \end{aligned}$$

Evaluate the determinant of the coefficient matrix.

$$\begin{vmatrix} 1 & 4 & 0 \\ 3 & 8 & 3 \\ 0 & 2 & 1 \end{vmatrix} = (8+0+0)-(0+6+12) = -10$$

SOLUTION

Apply cramer's rule since determinant is not zero.

$$C = \frac{\begin{vmatrix} 16 & 4 & 0 \\ 92 & 8 & 3 \\ 18 & 2 & 1 \end{vmatrix}}{-10} = \frac{-120}{-10} = 12$$

Atomic weight of carbon = 12

$$H = \frac{\begin{vmatrix} 1 & 16 & 0 \\ 3 & 92 & 3 \\ 0 & 18 & 1 \end{vmatrix}}{-10} = \frac{-10}{-10} = 1$$

Atomic weight of hydrogen =1

$$O = \frac{\begin{vmatrix} 1 & 4 & 16 \\ 3 & 8 & 92 \\ 0 & 2 & 18 \end{vmatrix}}{-10} = \frac{-160}{-10} = 16$$

Atomic weight of oxygen =16

Thanks