

Introduction to Differential Equations

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ordinary differential equations

Definition:

A differential equation is an equation containing an unknown function and its derivatives.

Examples:.

$$1. \quad \frac{dy}{dx} = 2x + 3$$

$$2. \quad \frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + ay = 0$$

$$3. \quad \frac{d^3 y}{dx^3} + \left(\frac{dy}{dx} \right)^4 + 6y = 3$$

y is dependent variable and x is independent variable,
and these are ordinary differential equations

Partial Differential Equation

Examples:

$$1. \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

u is **dependent variable** and x and y are **independent variables**, and is **partial differential equation**.

$$2. \quad \frac{\partial^4 u}{\partial x^4} + \frac{\partial^4 u}{\partial t^4} = 0$$

$$3. \quad \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} - \frac{\partial u}{\partial t}$$

u is **dependent variable** and x and t are **independent variables**

Order of Differential Equation

The **order** of the differential equation is order of the highest derivative in the differential equation.

Differential Equation

ORDER

$$\frac{dy}{dx} = 2x + 3$$

1

$$\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 9y = 0$$

2

$$\frac{d^3 y}{dx^3} + \left(\frac{dy}{dx} \right)^4 + 6y = 3$$

3

Degree of Differential Equation

The **degree** of a differential equation is **power of the highest order derivative term in the differential equation.**

Differential Equation

Degree

$$\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + ay = 0$$

1

$$\frac{d^3 y}{dx^3} + \left(\frac{dy}{dx} \right)^4 + 6y = 3$$

1

$$\left(\frac{d^2 y}{dx^2} \right)^3 + \left(\frac{dy}{dx} \right)^5 + 3 = 0$$

3

Linear Differential Equation

A differential equation is **linear**, if

1. dependent variable and its derivatives are of degree one,
2. coefficients of a term does not depend upon dependent variable.

Example: 1. $\frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + 9y = 0.$

is linear.

Example: 2. $\frac{d^3 y}{dx^3} + \left(\frac{dy}{dx}\right)^4 + 6y = 3$

is non - linear because in **2nd term** is not of degree one.

Example: 3.

$$x^2 \frac{d^2 y}{dx^2} + y \frac{dy}{dx} = x^3$$

is non - linear because in **2nd term** coefficient depends on y .

Example: 4.

$$\frac{dy}{dx} = \sin y$$

is non - linear because $\sin y = y - \frac{y^3}{3!} + \dots$ is non - linear

9. Table 1. Classify each differential equation

No	Differential Equations	Ordinary or Partial	Linear or nonlinear	Order	Degree	Independent variables	Dependent variables
1.	$y' = x + 6y$						
2.	$y'' = 4y + y^3$						
3.	$\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} - 2y = x^3$						
4.	$y'' + 2xy' + 4y = \cos 2x$						
5.	$\frac{dy}{dx} = \frac{x^2-1}{y+4}$						
6.	$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$						
7.	$\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2}$						

It is **Ordinary/partial** Differential equation of **order...** and of degree..., it is **linear / non linear**, with **independent** variable..., and **dependent** variable....

1st – order differential equation

1. Derivative form:

$$a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

2. Differential form:

$$(1+x)dy - ydx = 0$$

3. General form:

$$\frac{dy}{dx} = f(x, y) \quad \text{or} \quad f\left(x, y, \frac{dy}{dx}\right) = 0.$$

First Order Ordinary Differential equation

$$f\left(x, y, \frac{dy}{dx}\right) = 0.$$

$$\frac{dy}{dx} = f(x, y)$$

$$M(x, y)dx + N(x, y)dy = 0$$

$$a_1(x) \frac{dy}{dx} + a_0(x)y = g(x)$$

$$a_1(x)y' + a_0(x)y = g(x)$$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Derivative form

Differential form

Standard form

Standard form

First order linear differential equation form

Second order Ordinary Differential Equation

$$f\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}\right) = 0.$$

$$\frac{d^2y}{dx^2} = f\left(x, y, \frac{dy}{dx}\right)$$

$$a_2(x) \frac{d^2y}{dx^2} + a_1(x) \frac{dy}{dx} + a_0(x)y = g(x)$$

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = g(x)$$

nth – order linear differential equation

1. nth – order linear differential equation with constant coefficients.

$$a_n \frac{d^n y}{dx^n} + a_{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_2 \frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_0 y = g(x)$$

2. nth – order linear differential equation with variable coefficients

$$a_n(x) \frac{dy}{dx} + a_{n-1}(x) \frac{d^{n-1} y}{dx^n} + \dots + a_2(x) \frac{d^2 y}{dx^2} + a_1(x) \frac{dy}{dx} + a_0(x) y = g(x)$$

Solution of Differential Equation

Examples

$y=3x+c_1$, is solution of the 1st order differential equation $\frac{dy}{dx} = 3$ c_1 is arbitrary constant. As is solution of the differential equation for every value of c_1 , hence it is known as general solution.

Examples

$$y' = \sin(x) \Rightarrow y = -\cos(x) + C$$

$$y'' = 6x + e^x \Rightarrow y' = 3x^2 + e^x + C_1 \Rightarrow y = x^3 + e^x + C_1x + C_2$$

Observe that the set of solutions to the above 1st order equation has 1 parameter, while the solutions to the above 2nd order equation depend on two parameters.

Families of Solutions

Example

$$9yy' + 4x = 0$$

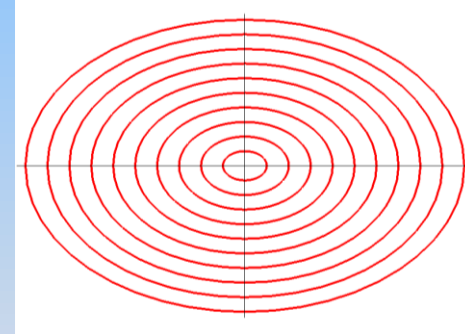
Solution

$$\int (9yy' + 4x) dx = C_1 \Rightarrow \int 9y(x)y'(x)dx + \int 4x dx = C_1$$

$$\Rightarrow \int 9y dy + 2x^2 = C_1 \Rightarrow \frac{9y^2}{2} + 2x^2 = C_1 \Rightarrow 9y^2 + 4x^2 = 2C_1$$

This yields $\frac{y^2}{4} + \frac{x^2}{9} = C$ where $C = \frac{C_1}{18}$.

Observe that given any point (x_0, y_0) , there is a unique solution curve of the above equation which curve goes through the given point.



The solution is a family of ellipses.

Origin of Differential Equations Solution

1. Geometric Origin

1. For the family of straight lines

$y = c_1x + c_2$ the differential equation is

$$\frac{d^2 y}{dx^2} = 0$$

2. For the family of curves

A. $y = ce^{\frac{x^2}{2}}$ the differential equation is $\frac{dy}{dx} = xy$

B. $y = c_1e^{2x} + c_2e^{-3x}$
the differential equation is $\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 6y = 0$

Physical Origin

1. Free falling stone $\frac{d^2 s}{dt^2} = -g$

where s is distance or height and
 g is acceleration due to gravity.

2. Spring vertical displacement $m \frac{d^2 y}{dt^2} = -ky$

where y is displacement,

m is mass and
 k is spring constant

3. RLC – circuit, Kirchoff 's Second Law

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{1}{c} q = E$$

q is charge on
capacitor,

L is inductance,

c is capacitance.

R is resistance and

E is voltage

Physical Origin

1. Newton's Law of Cooling

$$\frac{dT}{dt} = \kappa (T - T_s)$$

where $\frac{dT}{dt}$ is rate of cooling of the liquid,
 $T - T_s$ is temperature difference between the liquid 'T'
and its surrounding T_s

2. Growth and Decay

$$\frac{dy}{dt} = \kappa y$$

y is the quantity present at any time

THANKING YOU